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Parameter Optimisation for the Behaviour of Elastic Models over Time.

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Abstract. Optimisation of parameters for elastic models is essential for comparison or finding equivalent behaviour of elastic models when parameters cannot simply be transferred or converted. This is the case with a large range of commonly used elastic models. In this paper we present a general method that will optimise parameters based on the behaviour of the elastic models over time.

1. Introduction

In surgical simulation we use elastic models to calculate the deformations of the tissue in response to forces from virtual surgical instruments [1]. The actual behaviour of a given elastic model is determined by the material parameters. Parameters can often not simply be transferred from one model to another.
However, in cases where we would like to compare or optimise the behaviour of one elastic model to another, we need to find optimal parameters. Optimality is defined as the parameters that will make the models behave most alike. Such a method is presented in this proceeding.

2. Related Work

Previous research [2] has informally validated that Spring Mass and FEM behave alike, but there is a lack of a formal experimental validation. For this we will need optimal parameters of all elastic models.
Previous models for parameter optimisation for elastic objects [3-4] have focused on the equilibrium or time until equilibrium and not the time dependant properties of the elastic models, even though the behaviour over time is often of more importance in real-time applications.
Previous models have also not focused on the full range of parameters for e.g. spring mass based models [4] or have used manual optimisation of parameters [3].

3. Strategy of Optimisation

We use an Evolutionary Algorithm (EA) [5] approach to the optimisation. That is, we evolve a set of chromosomes through natural selection. An EA is especially useful when the properties of the fitness landscape is unknown and as such is well suited for this kind of

parameter optimisation - especially for heuristic elastic models. Chromosomes are defined corresponding to parameters of their related elastic model. E.g. the chromosome for a standard Spring Mass model could be given by step size, spring stiffness and damping.
We will define a reference elastic model, being the model we would like to transfer behaviour from, and a target elastic model, which is the model we would like to find parameters for.
A complete optimisation must define geometry and a set of interactions. These can be chosen from real use situations, to let the elastic models adapt optimally to the real use situation.
The problem of finding parameters to optimise speed of convergence and the accuracy of the equilibrium is actually a multi-objective optimization. The accuracy of the equilibrium and speed of convergence depend on each other. The tradeoff between the two parameters in the fitness evaluation is implicit in the interactions. It depends on the ratio of the time the tissue is interacted with and the time it is not.

4. Evaluation of Fitness

We will now look more closely at the fitness evaluation. The metric is based on the average of the distance between nodes paired in the two models. The nodes are paired through their position in space.
A fitness evaluation of chromosome *c* at one point in time *s*, with positions x^t indicating coordinates of target node *t* and x^r indicating coordinates of reference node *r*:

$$compare(t,r,s) = \sum_{i=1}^{N} \frac{\left\| x(s)_i^t - x(s)_i^r \right\|}{N}$$

The important contribution of this paper is the recognition that the behaviour over time is of importance to real-time applications. The actual fitness is therefore the average of the average distances of nodal points over some period of time. The reference model and target model are compared at a set of points in time $s \in S$.

$$fitness(t,r) = \sum_{s \in S} \frac{compare(t,r,s)}{size(S)}$$

It is important to recognize the fact that we are not trying to find parameters that will make the elastic models run faster. The speed of the elastic models is an integral part of the definition of the elastic models.

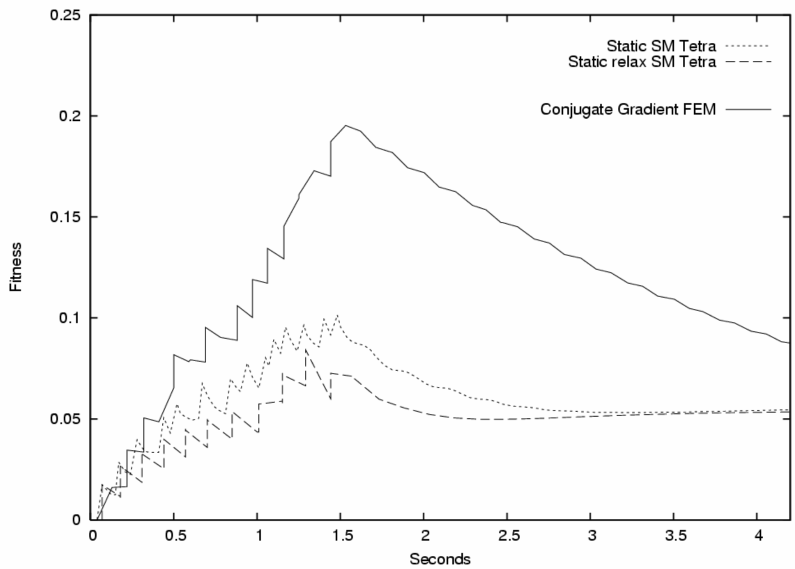

Figure 1. Convergence of the elastic models in comparison to static FEM.

5. Using the Approach to Compare Elastic Models

We have used the optimisation method described to optimise elastic models to a pre-calculated static linear finite element based deformation. The target elastic models for the optimisation are quasi-static Spring Mass (SM) [6] with and without relaxation [7] and a conjugate gradient solution of a Finite Element (CG FEM) [8] based deformation.
In figure 1 we compare the elastic models with optimal parameters as found with the method described above. At 0 seconds the stretch is started and at about 1.5 seconds it is stopped. We can see that the CG FEM does not mimic the reference model as closely as the SM models for the first 4-5 seconds. But because the CG FEM and the reference solve the same equations, the CG FEM will converge to an actual 0 in fitness.
Most importantly the preliminary tests shows that SM based algorithms generally converge faster than the CG FEM within the first few seconds. However the SM based algorithms cannot reach the absolute equilibrium.

6. Conclusions and Future Work

The EA for parameter optimisation was developed and tested on a range of elastic objects. We have created a parameter optimisation that can explicitly optimise for behaviour over time.
The preliminary comparison suggests that CG FEM and SM could be combined in a hybrid solution to achieve a fast initial convergence through SM but an accurate equilibrium through the CG FEM.
The next step in the research is to enlarge the set of test-interactions, and include a wider range of elastic models for comparison.

7. References

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